Find the general solution of the differential equation $x^2y'' - 4xy' + 6y = x^3 \ln x$. **SCORE: ____ / 7 PTS**

$r^2 - 5r + 6 = 0 \rightarrow r = 2, 3$

$y_h = C_1x^2 + C_2x^3$

$g = \frac{x^3 \ln x}{x^2} = x \ln x$ (1)

$\begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix} = 3x^4 - 2x^4 = x^4$

$y_p = -x^2 \int \frac{(x \ln x)x^3}{x^4} dx + x^3 \int \frac{(x \ln x)x^2}{x^4} dx$

$= -x^2 \int \ln x \, dx + x^3 \int \frac{\ln x}{x} dx$

$= -x^2(x \ln x - x) + x^3\left(\frac{1}{2}(\ln x)^2\right)$ (1)

$= -x^3 \ln x + x^3 + \frac{1}{2}x^3(\ln x)^2$

$y = -x^3 \ln x + \frac{1}{2}x^3(\ln x)^2 + C_1x^2 + C_2x^3$

ALL ITEMS WORTH (1/2) POINT EACH UNLESS OTHERWISE NOTED

Find the form of the particular solution of the differential equation $y^{(4)} - 6y''' + 10y'' = x^2 + xe^{3x} \sin x$. **SCORE: ____ / 5 PTS**

$r^4 - 6r^3 + 10r^2 = 0 \rightarrow r^2(r^2 - 6r + 10) = 0 \rightarrow r = 0, 0, 3 \pm i$

$y_h = C_1 + C_2x + C_3e^{3x}\cos x + C_4e^{3x}\sin x$

$y_p = (Ax^2 + Bx + C)x^2 + ((Dx + E)e^{3x}\cos x + (Fx + G)e^{3x}\sin x)x$

(1) (1) (1)

ALL ITEMS WORTH (1/2) POINT EACH UNLESS OTHERWISE NOTED

Find the general solution of the differential equation $y'' + 4y' + 4y = e^{-2x} - 24\sin 2x$.

SCORE: _____ / 18 PTS

$r^2+4r+4=0 \rightarrow r=-2,-2$ (1)

$y_h = C_1e^{-2x} + C_2xe^{-2x}$ (2)

$y_p = Ax^2e^{-2x} + B\sin 2x + C\cos 2x$ (2)

$y_p' = (-2Ax^2+2Ax)e^{-2x} - 2C\sin 2x + 2B\cos 2x$ (2)

$y_p'' = (4Ax^2-8Ax+2A)e^{-2x} - 4B\sin 2x - 4C\cos 2x$ (3)

$+4y_p' \quad +(-8Ax^2+8Ax)e^{-2x} - 8C\sin 2x + 8B\cos 2x$

$+4y_p \quad +(4Ax^2)e^{-2x} + 4B\sin 2x + 4C\cos 2x$

$= \quad 2Ae^{-2x} - 8C\sin 2x + 8B\cos 2x$ (3)

$2A=1 \quad -8C=-24 \quad 8B=0$

$A=\frac{1}{2} \quad C=3$

$y = \frac{1}{2}x^2e^{-2x} + 3\cos 2x + C_1e^{-2x} + C_2xe^{-2x}$ (1)

(2) (2)